Physics: Exam n°3

Thursday 12 March 2014 **Duration : 1h45**

Approximate Marking Scheme*: *Exercise 1: 11 points, Exercise 2: 9 points.

Exercise 1: The didgeridoo, a traditional musical instrument

The Aborigines of northern Australia play a form of music, so deep that it sounds as if it comes from the Earth itself, to accompany chants during celebrations and rituals. Their principal instrument is the didgeridoo. It is claimed that it dates from the Stone Age, approximately 20,000 to 60,000 years ago. It is made from the stem of a young eucalyptus tree, which has been hollowed out by termites while the tree is alive. A rim of beeswax is sometimes applied to the mouthpiece through which one blows producing sounds which are strangely contemporary. When well played, the instrument emits a strange low frequency drone with the possibility of varying the rhythm, timbres and harmonics.

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Short glossary for the layperson:
Pitch of a sound: corresponds to the frequency of vibration of the sound wave.
Richness of a sound: sometimes described in terms of a sum of a number of distinct frequencies present. This contributes in part to the timbre of a musical instrument which enables one to distinguish the sounds of one musical instrument to another.
Note: $C = 340\,m.s^{-1}$

Generally, a didgeridoo is cylindrical, of 5 or 6 cm diameter and 1 m to 2 m in length.

Part A - *To obtain the basic sound, "the drone", the musician produces a monotone vibration with his/her lips; the tongue is immobile at the front of the mouth (assume this end is closed). We can therefore represent the instrument as a resonant tube of length, L_1, closed at one end and open at the other, with an axis of revolution (Oz).*

1. Explain, in a few lines, which type of wave will form in this musical instrument. Describe the physical phenomena involved without presenting any mathematical relationships.

2. For a harmonic excitation, determine, with the aid of the boundary conditions, the particle velocity $\underset{\sim}{v}(z,t)$ and the overpressure $\underset{\sim}{p}(z,t)$ at each point within the resonant tube. Comment on your answer.

NB: Give a detailed demonstration. The results should be expressed as a function of, among other things, the complex amplitude of the overpressure of the incident wave $\underset{\sim}{p_i}$, the acoustic impedance Z, and the angular wave number k.

3. Establish the relationship which expresses the resonant frequencies of this didgeridoo, as a function of its length L_1 and the velocity of sound C.

4. *A recording of the basic sound of a didgeridoo (the "drone") is provided on figure 1.a.*
   From the waveform on figure 1.a, determine the fundamental frequency, stating clearly the method used to make the measure. Deduce the length L_1 of the instrument.

5. What is the minimum length of a flute (a resonant tube open at both extremities) to produce a note of the same pitch?
   NB: one does not need to determine again the expression for $\underset{\sim}{v}(z,t)$ nor for $\underset{\sim}{p}(z,t)$ to answer this question.

Part B – *We will now study a second didgeridoo, of length L_2. The waveform for this instrument and the associated frequency spectrum are presented on figures 2.a and 2.b.*

1. Determine the fundamental frequency of this second instrument and its length L_2.

2. With the aid of the spectra for the two instruments (figures 1.b et 2.b), compare the sounds they produce. In your opinion, how can one produce two different sounds with the same instrument?

3. On the spectrum on figure 2b, determine the order of the harmonic with the next largest amplitude to that of the fundamental.
   Is this result coherent with the expression derived in question A.3?
   Give a physical explanation of this result.

4. Sketch the overpressure wave, which corresponds to the harmonic referred to in the previous question, as a function of position within the tube and at several suitably chosen instants.
   NB: The points and characteristic lengths should be clearly identified on the sketch.

Exercise 2: Some aspects of the circulatory system

A blood vessel can be modelled by an elastic and deformable tube, of infinite length along the Ox axis in which a fluid, i.e. blood (see figure 1), circulates, whose density at rest is ρ_0=1.0 kg/ℓ. We will assume in the following that all physical quantities depend only on the abscissa x and on time t. When the pressure wave is passing by, we denote p(x,t) the overpressure and v(x,t) the velocity of the fluid which is assumed to be oriented along Ox. Gravity effects are neglected. The elastic tube has a variable section S(x,t).

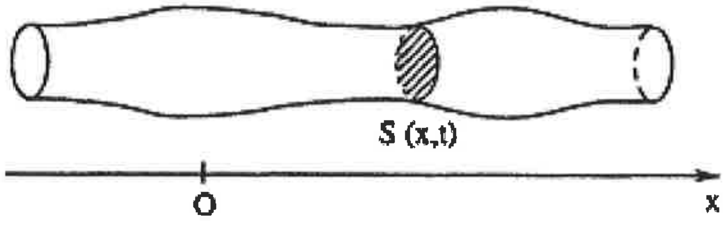

Figure1

1) *It can be established that the overpressure waves p verify the following equation:*

$$\frac{\partial^2 p}{\partial x^2} - \rho_0(D+\chi)\frac{\partial^2 p}{\partial t^2} = 0$$

where $D = \frac{1}{S}\frac{\partial S}{\partial P}$ *is the distensibility coefficient of the tube and* $\chi = \frac{1}{\rho}\frac{\partial \rho}{\partial P}$ *is the isentropic compressibility coefficient of the fluid. The value of* χ *is here* 5,0 10^{-10} Pa^{-1}.

Deduce the velocity c of the overpressure waves in blood (i) in the case of a blood vessel of distensibility D_m = 4.0.10^{-5} Pa^{-1} and (ii) in the case where the blood vessel is replaced by a metal tube of distensibility D_m = 1.0.10^{-11} Pa^{-1}. Comment upon the results.

2) *In the following we will neglect the compressibility* χ *of the fluid. The heart imposes to the blood vessel a volumetric flow rate Q(x,t)= S.v(x,t) which is assumed to be sinusoidal.* Explain, in one sentence, the meaning of this volumetric flow rate.

3) *The hydraulic impedance Z of the system is defined as the ratio of the overpressure p(x,t) to the volumetric flow rate Q(x,t). For a harmonic overpressure wave (of angular frequency ω) propagating along increasing x, we have: p=Z.Q.*

Knowing that: $\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$, prove that Z is equal to $\frac{\rho_0 c}{S}$.

Establish the relationship between p and Q for a wave propagating along decreasing x.

4) *The circulatory system has a branching structure. We will consider the first branching of the aorta, which starts at the heart, and bifurcates into the two iliac arteries; we can model this branching in terms of an artery of section s_1 which splits, at x=0, into the two finer arteries. The sum of the cross-sectional areas of the two arteries is s_2 (see Figure 2).* *<u>s_2 is smaller than s_1</u>. At x = 0, we have:*

- *Continuity of the volumetric flow rate Q,*
- *Continuity of the overpressures.*

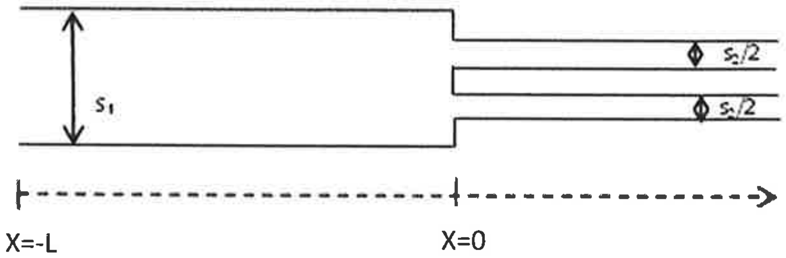

Figure 2

We consider a sinusoidal wave, generated at the start of the aorta, i.e. upstream with respect to the branching, whose overpressure in complex notation will be denoted $\underline{p_i}(x,t)$.

What happens at the branching and why?

Establish, <u>using a clear and rigorous demonstration</u>, that the coefficient of reflection $\underline{r}$ for the overpressures is $\underline{r} = \frac{s_1 - s_2}{s_1 + s_2}$. Deduce the coefficient of transmission $\underline{t}$ for the overpressures as a function of s_1 and s_2.

5) Show that the global hydraulic impedance, $\underline{I_h}$(-L) , defined as the ratio of the <u>total</u> overpressure over the <u>total</u> volumetric flow rate <u>at x = -L</u> verifies :

$$\underline{I_h(-\mathrm{L})} = Z_1 \frac{1 + i\frac{s_2}{s_1}\tan kL}{\frac{s_2}{s_1} + i\tan kL} \quad \text{with } Z_1 = \frac{\rho_0 c}{s_1}$$

6) *One can show that the <u>modulus</u> of this impedance has minima and maxima when kL is a multiple of $\pi/2$.*

Indicate, as a function of λ, for which values of L this modulus is maximum and for which values of L it is minimum. In both cases, give the value of $\underline{I_h}$(-L).

7) *For a human being, the length of the aorta is 1.25 m, the heart rate is 60 beats per minute, and the ratio of the cross sections s_2/s_1 is 0.4.*

Show that this corresponds to an ideal situation for the heart.

Annexes of Exercise n°1

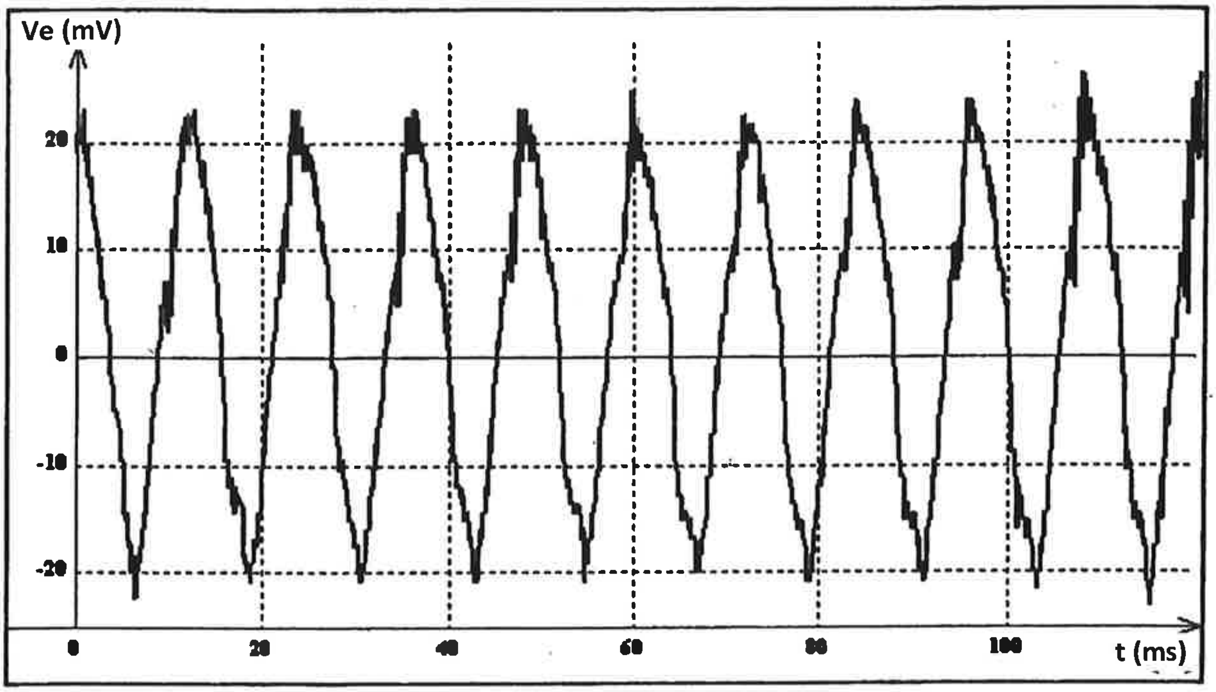

Figure 1.a – Waveform of a didgeridoo of length L_1. V_e represents the voltage measured at the output of the microphone.

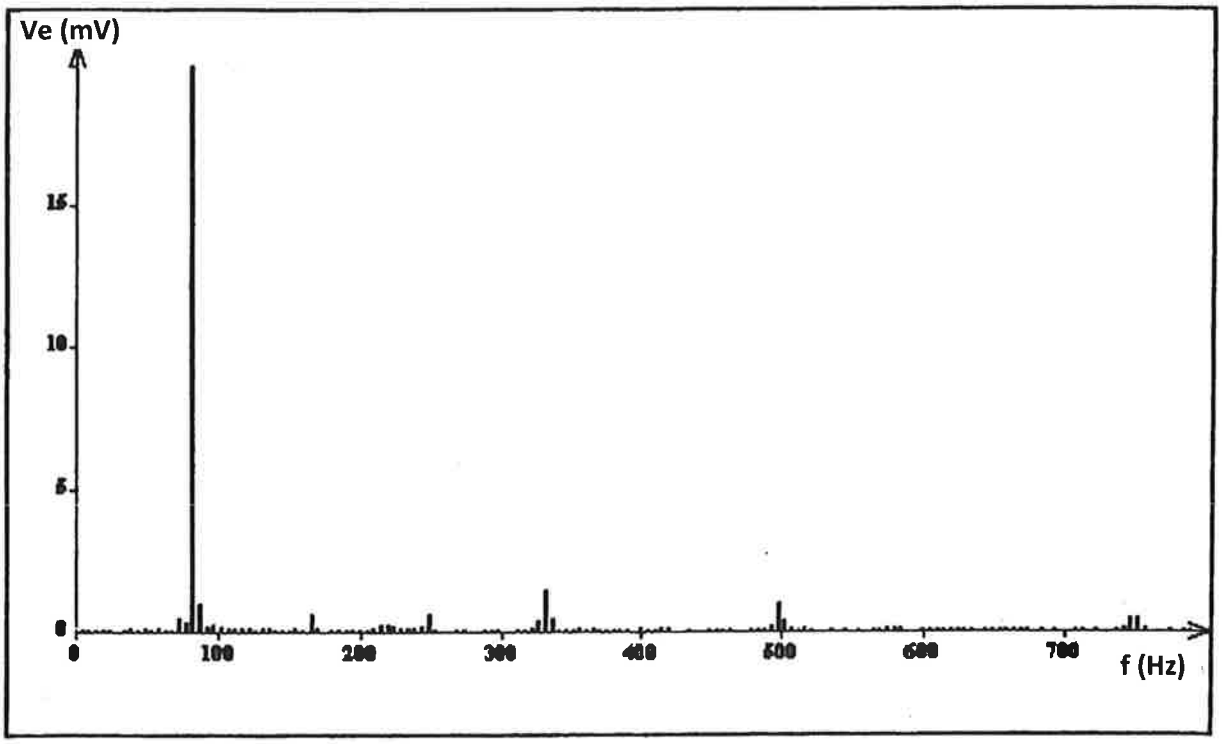

Figure 1.b – Frequency Spectrum of the waveform shown on figure 1.a.

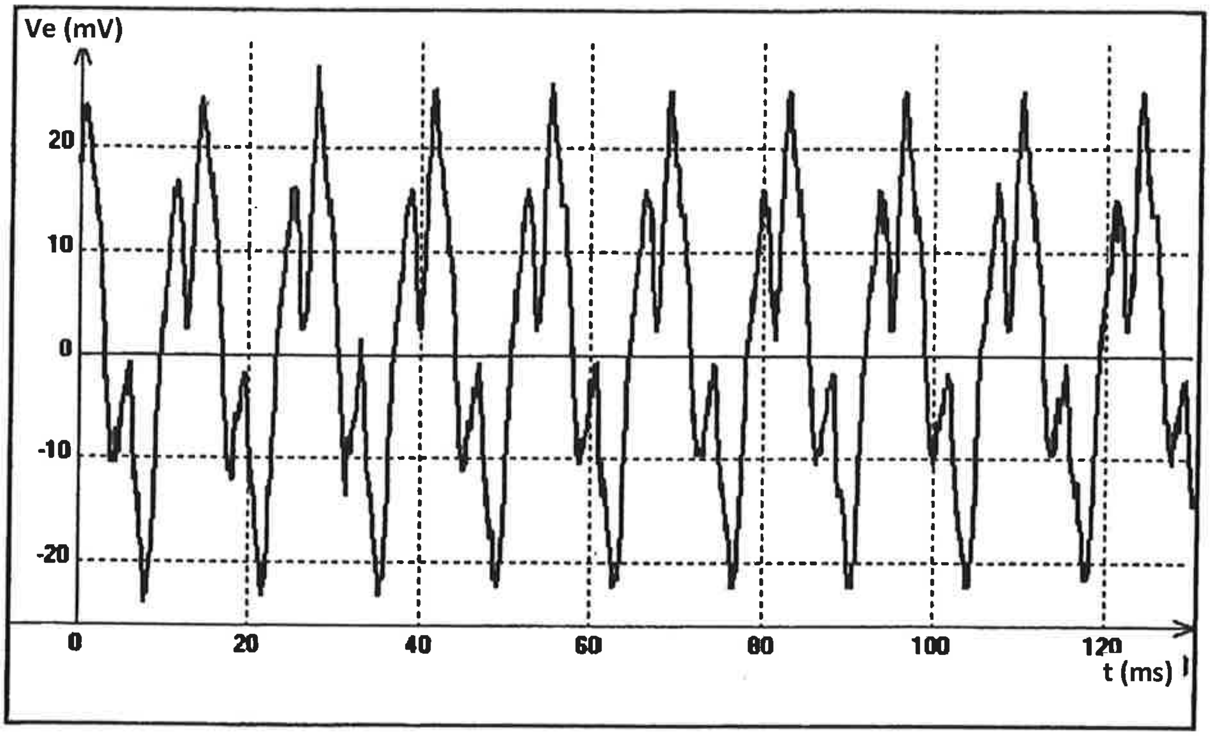

Figure 2.a – Waveform of a didgeridoo of length L_2

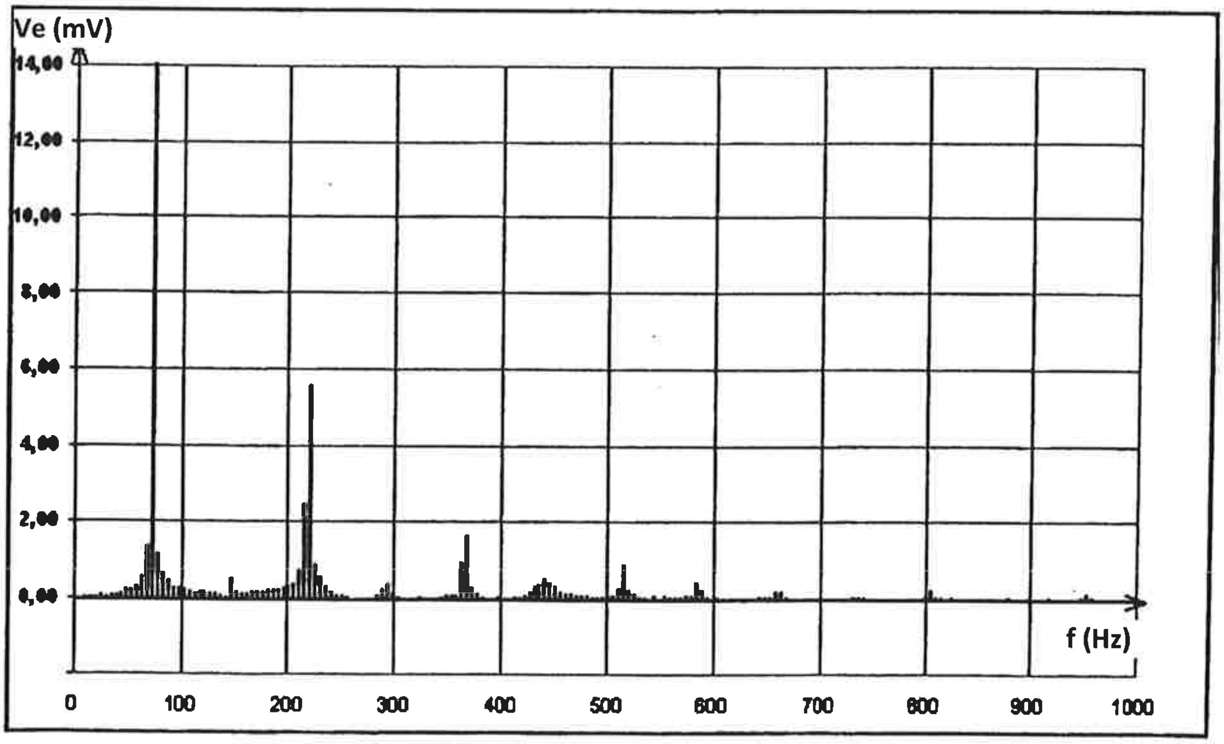

Figure 2.b – Frequency Spectrum of the waveform shown on figure 2.a.