Mathematical Tools for Engineering

SCAN 1st

Test 1

2 hours - No document – the classic notations are used

Part 1 – Complex numbers:

1- Find the $x+iy$ form of :

a) $\left(\frac{1+i}{1-i}\right)^4$ b) $\left(\frac{1-i}{\sqrt{2}}\right)^{40}$ c) $e^{(2n+1)\pi i}$, *n=0,1,...* integer

2- Find the absolute values of

a) $\frac{2-i}{3+i}$ b) $\left(\frac{z}{\overline{z}}\right)^5$ where z is any complex number c) $(-i)^i$

3- Considering the two complex numbers $z=3+4i$ and $w=2-i$, calculate and plot on the complex plane (Argand diagram)

a) $\frac{z}{w}$, b) $\overline{z}w+\overline{w}z$ c) w^2

4- Noticing that

a. $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$

b. $\cos\left(\frac{\pi}{3}\right)=\frac{1}{2};\ \sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2};\ \cos\left(\frac{\pi}{4}\right)=\sin\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

and using $e^{i\frac{\pi}{12}}$, find the exact value of $\tan\left(\frac{\pi}{12}\right)$.

Same question for $\tan\left(\frac{7\pi}{12}\right)$

5- Give all the complex cube roots of 8.

Part 2 – Vectors:

1- Consider points A and B of coordinates $A(-1,1,1), B(0,1,-2)$

a. Give a unit vector in the direction *AB*

b. Deduce the equations defining the line passing through *A* and *B*.

2- Consider the line through $B(3,2,5)$ and collinear with $\vec{u} = \vec{x} + \vec{y}$

a. Find the distance between point $O(0,0,0)$ and the line defined above

b. Find the shortest distance between line $(B,\vec{u})$ and line $(O,\vec{v})$ where $\vec{v} = \vec{x} + 2\vec{z}$

3 – Using vectors find the acute angle between two diagonals of a cube.

4 – Let $\vec{u}$ and $\vec{v}$ be unit vectors in the $(\vec{x},\vec{y})$ plane making positive angles α and β $(\beta > \alpha)$ with the positive $\vec{x}$ axis.

a) Find the coordinates of $\vec{u}$ and $\vec{v}$ in the coordinate system $(\vec{x},\vec{y})$

b) Calculate $\vec{u}.\vec{v}$ and deduce the expression of $\cos(\beta - \alpha)$ in terms of $\cos\alpha, \sin\alpha, \cos\beta$ and $\sin\beta$. *Precise explanations are required!*

c) Calculate $\vec{u} \times \vec{v}$ and deduce the expression of $\sin(\beta - \alpha)$