SCAN 2nd

20th November, 2014

Mechanics - Test 1

Duration: 1 hour 30
Authorised documents: personal formula sheets
Remarks: Parts A, B, C and D are independent and can be treated in any order.

A. Statics

An engine represented by solid 1 (rigid) in Figure 1 is fixed to a test bench (solid 0) at points A, B and C. The objective is to determine the reaction forces at A, B, C. The joint at A is assimilated to a spherical joint of centre A; that at B, to a sphere-in-cylinder (annular) joint of centre B and axis $\vec{y}$. Finally, the joint at C is assimilated to a sphere-on-plane joint (or point contact) of centre C and axis $\vec{z}$. All the joints are supposed to be perfect.

Engine 1 is submitted to a torque $C_r\vec{x}$ generated by a brake (not shown); its mass is M and its centre of mass is G.

In the coordinate system $(G, \vec{x}, \vec{y}, \vec{z})$ shown in Figure 1, one defines the following vectors:

$$\overrightarrow{AB} = \ell_{yB}\vec{y} \qquad \overrightarrow{AC} = \ell_{xC}\vec{x} + \ell_{yC}\vec{y} + \ell_{zC}\vec{z} \qquad \overrightarrow{AG} = \ell_{xG}\vec{x} + \ell_{yG}\vec{y} + \ell_{zG}\vec{z}$$

These coordinates are algebraic and can be either positive or negative.

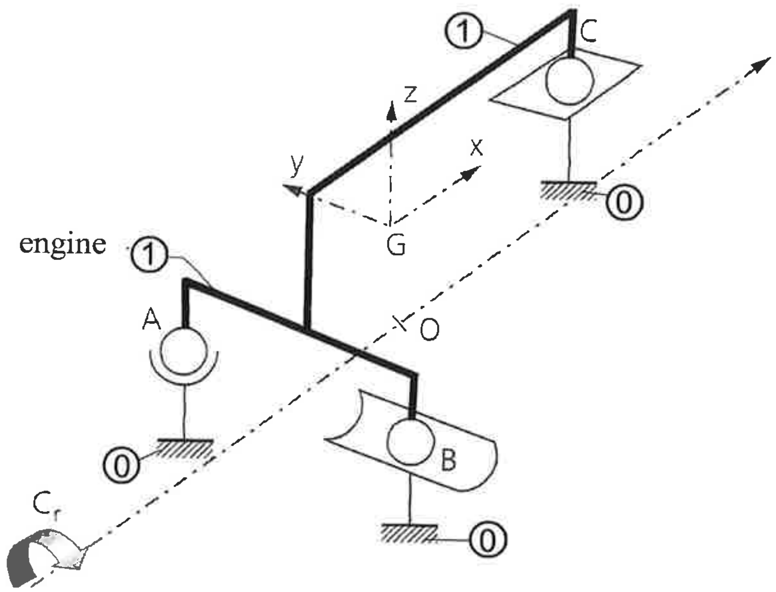

Figure 1. Engine on a test bench

- **A1.** Using the wrench formalism, give the list of all the external actions on the engine and develop the static equilibrium equations.
- **A2.** Determine the mechanical actions at A, B, C in terms of C_r, M and the coordinates of $\overrightarrow{AB}$, $\overrightarrow{AC}$ et $\overrightarrow{AG}$

B. Graphical statics

A clamping mechanism is shown in Figure 2. It comprises a ground (solid 0), three levers (1), (2), (3) and a jack (4). The objective is to block part (6) during machining.

The problem is planar (in plane $(O, \vec{x}, \vec{y})$ of the fixed coordinate system $(O, \vec{x}, \vec{y}, \vec{z})$)

Lever (1) is connected to ground (0) by a revolute joint of axis $(B, \vec{z})$.

Lever (3) is connected to ground (0) by a revolute joint of axis $(F, \vec{z})$.

Lever (2) is connected to levers (1) and (3) by revolute joints of axes $(C, \vec{z})$ and $(E, \vec{z})$ respectively.

Jack (4) is connected to levers (2) and (3) by revolute joints of axes $(D, \vec{z})$ and $(G, \vec{z})$ respectively.
Lever (1) is in point contact with part (6) at A.

The weights of all the parts are neglected and all the joints are supposed to be perfect. As a consequence, it will be assumed that the force between (1) and (6) is in the normal direction at A.

The system is in static equilibrium.

- **B1.** By graphical analysis, determine the force that jack (4) should exert on levers (2) and (3) in order to generate a clamping force of 1000 N between (1) and (6).

  Hints: Try to isolate solids or sets of solids (several solids) submitted to 2 or 3 forces only (two- or three-force members).
  The graphical constructions will be drawn on the present page which will be handed in with your paper (*please do not forget to indicate your name!*).
  Use the following scale: 10 mm for 100 N.
  The reasoning will be succinctly exposed.

- **B2.** Conclude on the efficiency of the clamping system.

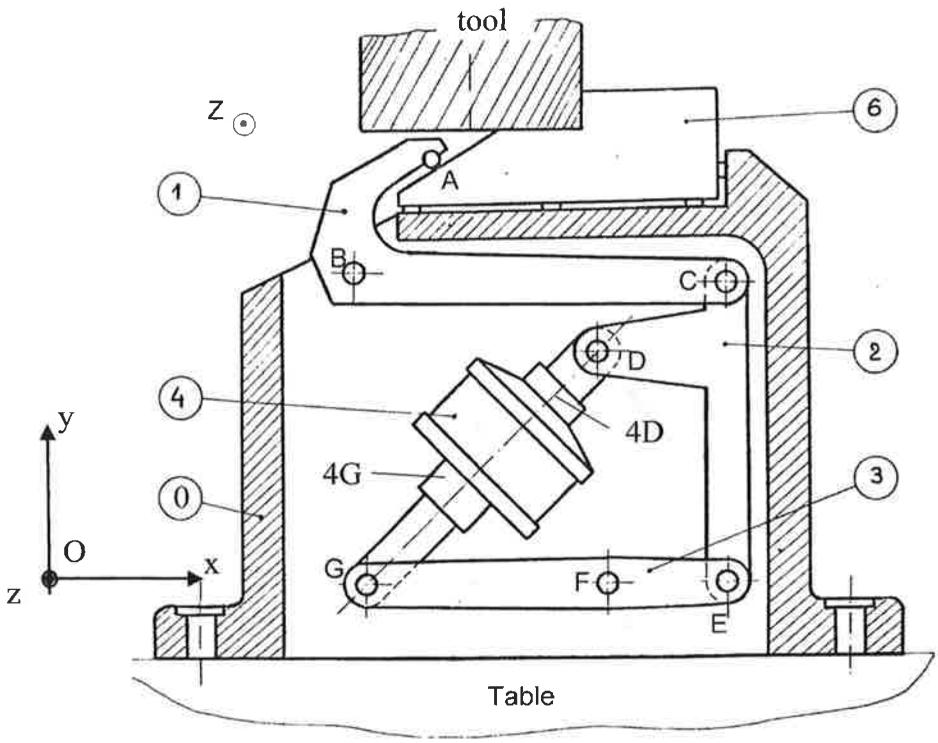

Figure 2. Clamping system

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C. Internal force wrench

The objective is to determine the internal forces and moments in some parts (beams) of the mechanism defined in part B. (Part C is independent of part B since the forces on the parts are different).
At any cross-section of a beam, N is the normal force (projection of the sum of the internal force wrench in the $\vec{x}$ direction), T represents the shear force (projection of the sum of the internal force wrench in the $\vec{y}$ direction) and M is the moment of bending (projection in the $\vec{z}$ direction of the moment of the internal force wrench at the centre of the cross section). The length of GF is 240 mm and that of FE is 120 mm. The axis GE of the beam is in the $\vec{x}$ direction. Points G, F and E are aligned.
In the configuration under study, part 3 (the beam) is submitted to the following forces:

At G : $\overrightarrow{G_{43}} = G_x\vec{x} + G_y\vec{y}$ at F : $\overrightarrow{F_{53}} = F_x\vec{x} + F_y\vec{y}$ at E : $\overrightarrow{E_{23}} = E_x\vec{x} + E_y\vec{y}$

The force components are: G_x=150 N, G_y=160N, E_x=30 N, E_y=320 N, F_x=-180 N, F_y=-480 N

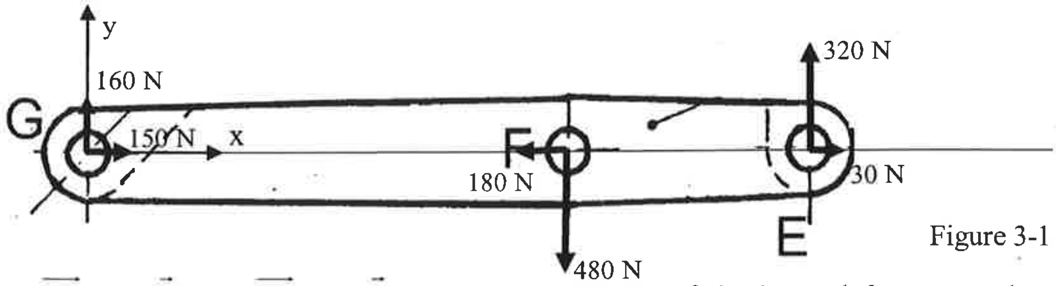

Figure 3-1

- **C1.** Denoting $\overrightarrow{GF} = \ell_{GF}\vec{x}$ and $\overrightarrow{FE} = \ell_{FE}\vec{x}$, give the expression of the internal force wrench coordinates in terms of the position x of the cross section along GE.
- **C2.** Represent on Figs. 3-2, 3-3 & 3-4 below, the variation of the normal force N(x), that of the shear force T(x) and that of the moment of bending M(x) between G and E. (x defines the position of the cross section along GE)

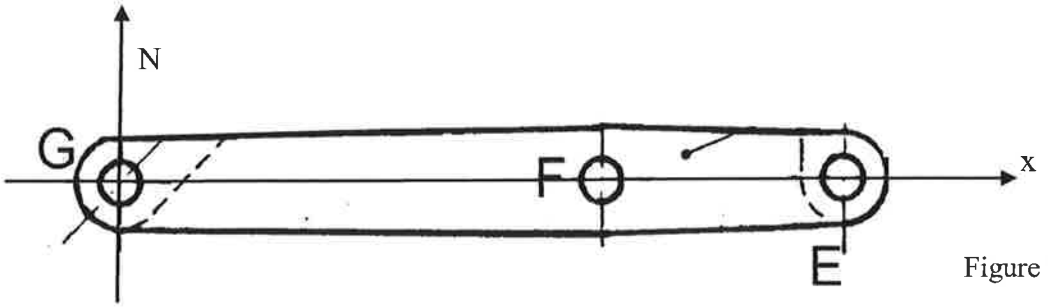

Figure 3-2

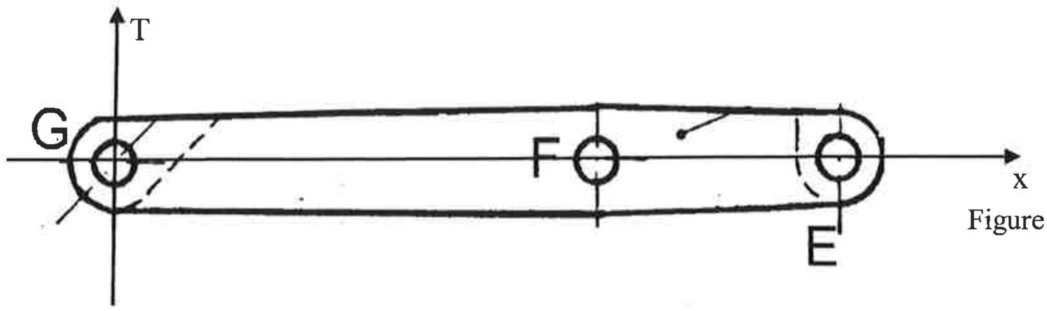

Figure 3-3

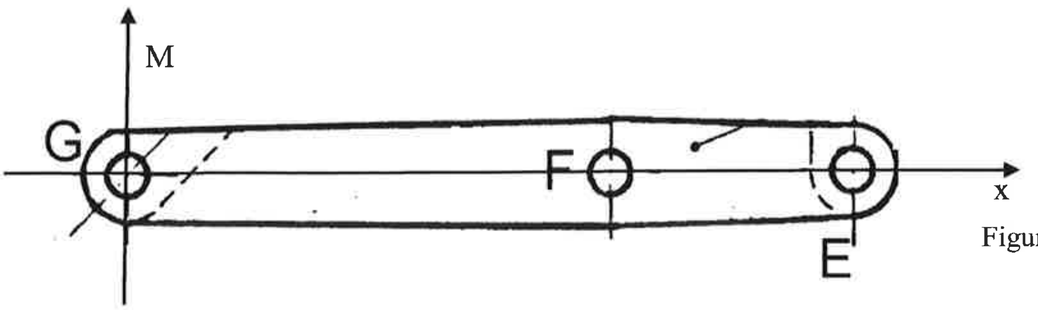

Figure 3-4

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D. Kinematic analysis

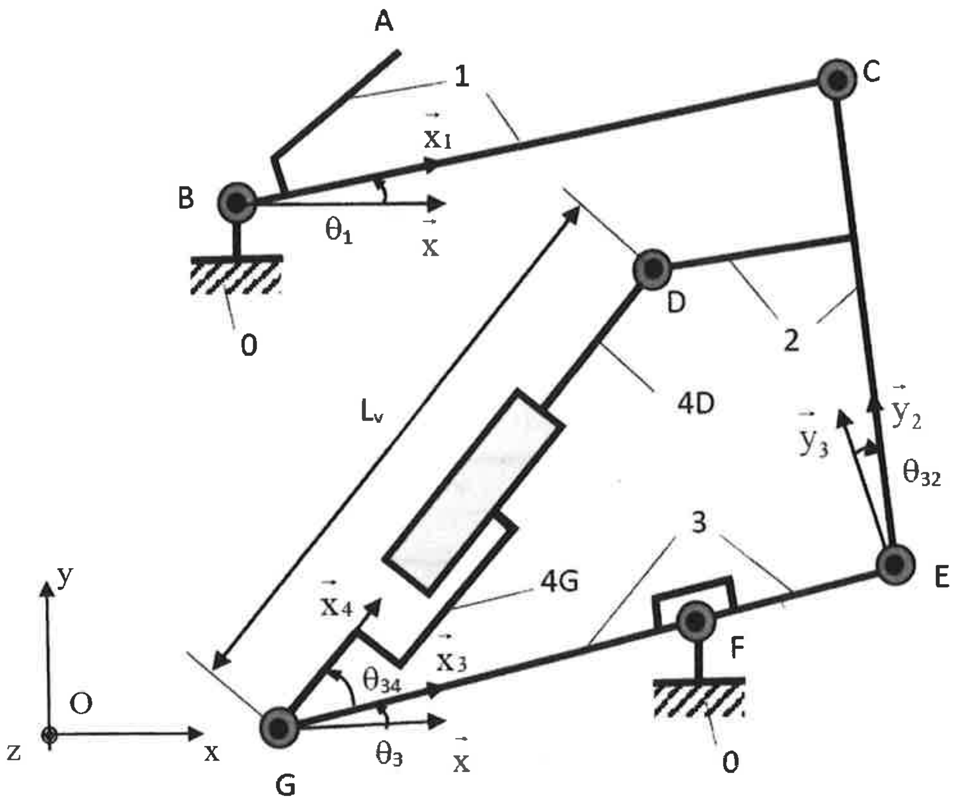

Figure 4. Kinematic model

In this section (part D), solid 6 is absent. The objective is to analyse the kinematics of the clamping system composed of the three levers (1), (2), (3), the jack (4D et 4G) and the ground (0). Jack (4) is considered as made of two parts: the upper part (4D) that includes point D and the lower part (4G) including point G (Figure 4). (4D) and (4G) are connected by a prismatic joint. All the parts are non-deformable (rigid).

The revolute joint axes at B, C, D, G, F, E are in the $\vec{z}$ direction. The axis of the prismatic joint between (4G) and (4D) of the jack is along $\overrightarrow{GD}$.

The problem is planar.

The following unit vectors are defined (see Figure 4):

$$\vec{x}_1 = \frac{\overrightarrow{BC}}{\|\overrightarrow{BC}\|} \qquad \vec{y}_2 = \frac{\overrightarrow{EC}}{\|\overrightarrow{EC}\|} \qquad \vec{x}_3 = \frac{\overrightarrow{GE}}{\|\overrightarrow{GE}\|} \qquad \vec{x}_4 = \frac{\overrightarrow{GD}}{\|\overrightarrow{GD}\|}$$

The lengths $\ell_{BC} = \|\overrightarrow{BC}\|$ $\ell_{EC} = \|\overrightarrow{EC}\|$, $\ell_{GE} = \|\overrightarrow{GE}\|$ and $\ell_{FE} = \|\overrightarrow{FE}\|$ are constant and known.

The positions of points B, D and F are known and one gives $\overrightarrow{BF} = x_{BF}\,\vec{x} + y_{BF}\,\vec{y}$ and $\overrightarrow{ED} = -a_D \vec{x}_2 + b_D \vec{y}_2$ (a_D and b_D positive).

The parameters for the joints 1/0, 2/3, 3/0, 4/3 and 4D/4G are defined as:

$$\theta_1 = (\vec{x}, \vec{x}_1) \qquad \theta_{32} = (\vec{y}_3, \vec{y}_2) \qquad \theta_3 = (\vec{x}, \vec{x}_3) \qquad \theta_{34} = (\vec{x}_3, \vec{x}_4) \qquad L_v = GD$$

- **D1.** Draw the change of basis diagrams and the graph of links.
- **D2.** Develop the constraint equations imposed by the revolute joints (with no parameter) at C and D.