16.25

Name : JAYLE **First name :** Théo **Group :** 61

Premier cycle -Première année
Scanfirst

28/02/2014

THERMODYNAMICS – Test n°1

Duration : 1h (advice : 10 minutes + 50 minutes)

Any document forbidden, any calculator allowed
The answers must be written on the paper

Data : $R = 8.314\ J.mol^{-1}.K^{-1}$ T(K) = T(°C) + 273 $1\,bar = 10^5\,Pa$

Elements	C	N	O
M ($g.mol^{-1}$)	12	14	16

Exercise 1 (*10 minutes, 5 points*)

Air is assumed an ideal gas consisting of 20 % dioxygen and 80 % dinitrogen. Under standard condition of pressure (1 bar) and at 25°C, **calculate** :

1. the mean molar mass of air $\overline{M}_{air} = x_{O_2} \cdot M_{O_2} + x_{N_2} M_{N_2}$ $\overline{M}_{air} = 0.2 \times 32 + 0.8 \times 28$ $\overline{M}_{air} = 28.8\ g.mol^{-1}$	1
2. the mass proportions of dinitrogen and dioxygen in air	0

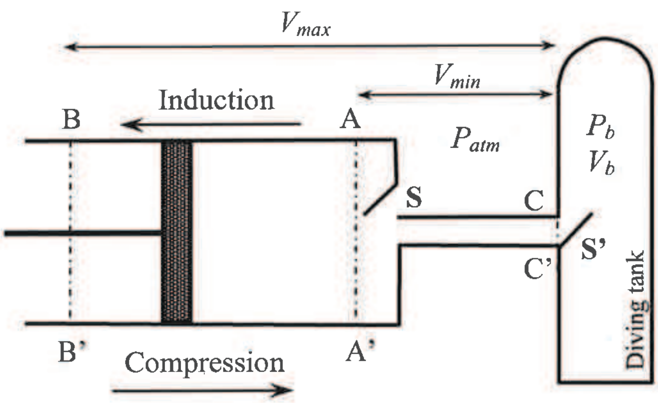
Some carbon dioxide (CO_2), assumed an ideal gas is introduced in a close container of fixed volume, already containing 1 mole of air in the standard condition of pressure (1 bar) and at 25°C, so that the final pressure reaches a value of 1.5 bar at constant temperature. Calculate :

3. the number of added CO_2 moles $PV = nRT$ or $\frac{n}{P} = VRT = cste$ $\frac{n_i}{P_i} = \frac{n_f}{P_f}$ $n_f = n_i \frac{P_f}{P_i}$ $n_f = n_{air} + n_{CO_2}$ $n_{CO_2} = n_f - n_{air}$ $\left(n_{i,air} \frac{P_f}{P_i} - n_{air}\right)$ $n_{CO_2} = 0.5\ mol$	1.5
4. the partial pressures of N_2, O_2 and CO_2 $P_i = x_i\ P_{tot}$ $x_{N_2} = \frac{0.8}{1.5} = 0.533$ $x_{O_2} = \frac{0.2}{1.5} = 0.133$ $x_{CO_2} = \frac{0.5}{1.5} = 0.33$ $P_{N_2} = x_{N_2} \cdot P_{tot} = 0.8\ bar = P_{N_2}$ $P_{O_2} = x_{O_2} \cdot P_{tot} = 0.2\ bar = P_{O_2}$ $P_{CO_2} = x_{CO_2} \cdot P_{tot}$ $P_{CO_2} = 0.5\ bar$	1.5
TOTAL ex. 1	4

Exercise 2 : filling of a diving tank (*50 minutes, 15 points*)

In order to fill a diving tank of rigid walls (cf figure bellow) defining a volume V_b, we use an air compressor made of a cylinder, of two valves **S** and **S'** and of a piston that can move without friction between the extreme positions AA' and BB'.

During the induction stage the valve **S** is open and **S'** closed; air enters in the cylinder at atmospheric pressure $P_{atm} = 1\,\text{bar}$. When the piston arrives in BB' the valve **S** is closed and the compression stage begins: the air in the cylinder is compressed. The valve **S** is closed and <u>when the pressure in the cylinder is higher than the pressure P_b in the tank, the valve **S'** opens</u>. When the piston is in AA', the volume of the compression chamber between AA' and CC' (S' closed) is V_{min} ; when the piston is in BB', this volume is V_{max}.



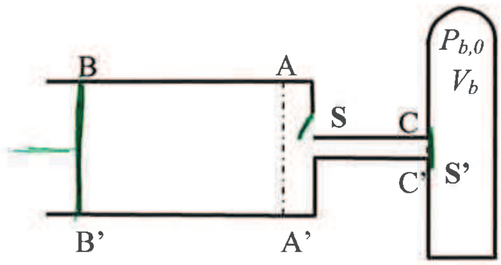
Air is submitted to <u>isothermal</u> processes (the temperature in the cylinder and in the tank are identical during the process and equal to the temperature T_{atm} of the atmosphere) ; the processes are assumed <u>quasi-static</u> and there is equilibrium at any time between the internal and external intensive quantities ; air is assumed an ideal gas.

I. First induction/compression cycle (1st forward/return).

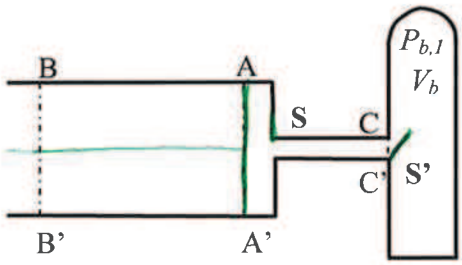
Since the compressor has not worked yet, the initial state of the **system** (= **air contained in the tank + volume between the piston and CC'**, written σ) is the following :

- tank : $P_{b,0} = P_{atm}$, T_{atm}
- cylinder : P_{atm} , T_{atm}, piston in AA'

5. Complete the schematic drawings below to specify the position of the piston and of the valve **S** and **S'** at the end of the induction and of the compression steps (use color line).



End of the induction step



End of the compression step

0.5

1

6. Explain why, during this first cycle, the valve **S closes** and valve **S' opens immediately** at the beginning of the compression step. At the beginning of the compression step, the air is pushed by the piston. The pressure in the cylinder increases, which pushes S in closed position since P_{atm} remains constant. The pressure in the cylinder gets higher than the pressure in the tanks as well, hence S' is pushed in open position.

0.5

7. Fill the table for the compression step with literal expressions :

	Initial state	Final state
Position of the piston	~~AA'~~ BB'	AA'
Volume occupied by **σ**	$V_{max} + V_b$	$V_{min} + V_b$
Pressure of **σ**	P_{atm}	$P_{b,1}$
Temperature of **σ**	T_{atm}	T_{atm}
Number of moles of **σ**	$\frac{P_{atm} \cdot (V_{max} + V_b)}{R \cdot T_{atm}}$	same as initial state

1.25

8. With the help of the above drawings and the table, noting that the number of moles must be constant during the compression step, establish the expression of the pressure $P_{b,1}$ in the tank at the end of the compression step, as function of P_{atm}, V_b, V_{min} and V_{max}.

$P_{b,1} V_f = n_f R T_f$ with $V_f = V_{min} + V_b$, $T_f = T_i = T_{atm}$, $n_f = n_i = \frac{P_{atm}(V_{max} + V_b)}{R \cdot T_{atm}}$

$P_{b,1} = \frac{P_{atm}(V_{max} + V_b) \; R \; T_{atm}}{R \cdot T_{atm}(V_{min} + V_b)}$ $\boxed{P_{b,1} = P_{atm} \cdot \frac{V_{max} + V_b}{V_{min} + V_b}}$

1.5

9. Establish the expression of the work transferred by the piston to the system **σ** during the first compression, as function of P_{atm}, V_b, V_{min} and V_{max}.

$\delta W = -P_{ext} \, dV = -P \, dV = -nRT \frac{dV}{V}$

$W = -\int_{V_{max}+V_b}^{V_{min}+V_b} nRT \frac{dV}{V} = nRT \int_{V_{min}+V_b}^{V_{max}+V_b} \frac{dV}{V} = \boxed{nRT \ln\left(\frac{V_{max} + V_b}{V_{min} + V_b}\right) = W}$

1

Moreover, at initial state $P_{atm} \cdot (V_{max} + V_b) = nRT$

Hence $\boxed{W = P_{atm}(V_{max} + V_b) \cdot \ln \frac{V_{max} + V_b}{V_{min} + V_b}}$

1

10. Establish the expression of the variation of the number of moles Δn contained <u>in the tank</u> during the first forward/return cycle.

<u>In the tank</u>:

(i) P_{atm}, T_{atm}, V_b hence $n_i = \frac{P_{atm} V_b}{R T_{atm}}$

(f) $P_{b,1}$, T_{atm}, V_b hence $n_f = \frac{P_{b,1} \cdot V_b}{R T_{atm}}$

$\boxed{\Delta n = \frac{V_b (P_{b,1} - P_{atm})}{R T_{atm}}}$

1

11. Numerical calculations ($V_b = 5\,\text{L}$, $V_{max} = 2\,\text{L}$, $V_{min} = 0.02\,\text{L}$, $T_{atm} = 20°\text{C}$, $P_{atm} = 1\,\text{bar}$) :

$P_{b,1}$ (in bar) = $1 \times \frac{5+2}{5+0.02} = 1.39$ bar $= P_{b,1}$ — 0.5

$W = 1(2+5) \cdot \ln\left(\frac{5+2}{5+0.02}\right) = 2.33$ J $= W$ units — 0.25

$\Delta n = \frac{5(1.39 - 1)}{8.314 \times 293} = 8.0 \times 10^{-4}$ mol $= \Delta n$ — 0.25

II. Following cycles.

The compressor has worked for i cycles. We consider that at a given time t the valve **S** is opened while the valve **S'** is closed. The state of the system is then :

- tank : $P_{b,i}$ $(P_{b,i} > P_{b,1})$, T_{atm}

- cylinder : P_{atm}, T_{atm}, piston in AA'

12. The piston works an additional induction/compression cycle. Conducting the same reflection than in question 8, find the volume of air V in the cylinder (between the piston position and CC'), as function of V_{max}, P_{atm} and $P_{b,i}$ when the valves S' opens. The piston goes to BB' (step int) and the it is pushed up to the point where S' opens, i.e., $P_{cylinder} = P_{b,i}$. (int) in the cylinder : V_{max}, T_{atm}, P_{atm}, n_i with $n_i = \frac{P_{atm} \cdot V_{max}}{R T_{atm}}$ When S' opens : in the cylinder : V, T_{atm}, $P_{b,i}$, n_i hence $V = \frac{n_i R T_{atm}}{P_{b,i}} = \frac{P_{atm} \cdot R \cdot T_{atm} \cdot V_{max}}{P_{b,i} \cdot R T_{atm}} = \frac{P_{atm} \cdot V_{max}}{P_{b,i}} = V$	1.5
13. Calculate the pressure $P_{b,i+1}$ in the tank at the end of this process (i.e when the piston is in AA'), as function of $P_{b,i}$, V_b, P_{atm}, V_{min} and V_{max}. $P_{b,i+1} = n_{i+1} \cdot \frac{RT}{V}$ $n_{i+1} =$ $P_{b,i}(V + V_b) = P_{b,i+1} \cdot (V_{min} + V_b)$ $P_{b,i+1} = P_{b,i} \frac{V+V_b}{V_{min}+V_b} = P_{b,i} \frac{V_b + P_{atm} + V_{max}}{V_b + V_{min}}$	0
14. We therefore obtain : $\Delta P_b = P_{b,i+1} - P_{b,i} = \frac{P_{atm} \cdot V_{max} - P_{b,i} \cdot V_{min}}{V_b + V_{min}}$. When does not the valve S' open any more ? It does not open when $\Delta P_b = 0$ i.e., $\frac{P_{atm} \cdot V_{max} - P_{b,i} \cdot V_{min}}{V_b + V_{min}} = 0$ $P_{atm} \cdot V_{max} = P_{b,i} V_{min}$ $P_{b,i} = \frac{P_{atm} \cdot V_{max}}{V_{min}}$	1
15. Establish the expression of the maximum pressure P_b^{max} reached in the tank as function of P_{atm}, V_{min} and V_{max} and calculate P_b^{max} in bar using the numerical data of question 11. According to the previous question $P_b^{max} = P_{atm} \frac{V_{max}}{V_{min}}$ $P_b^{max} = 1 \times \frac{2}{0.02}$ $P_b^{max} = 100$ bar	1
TOTAL exercise 2	12.25
Total (global)	