Thursday 27th March, 2014

Mechanics – TEST 3

The planar model below is used to study the rolling of a ship carrying a load by a crane such as the seaweed harvester (*goémonier)* shown in the photo.

Seaweed harvester in operation © Yves Gladu

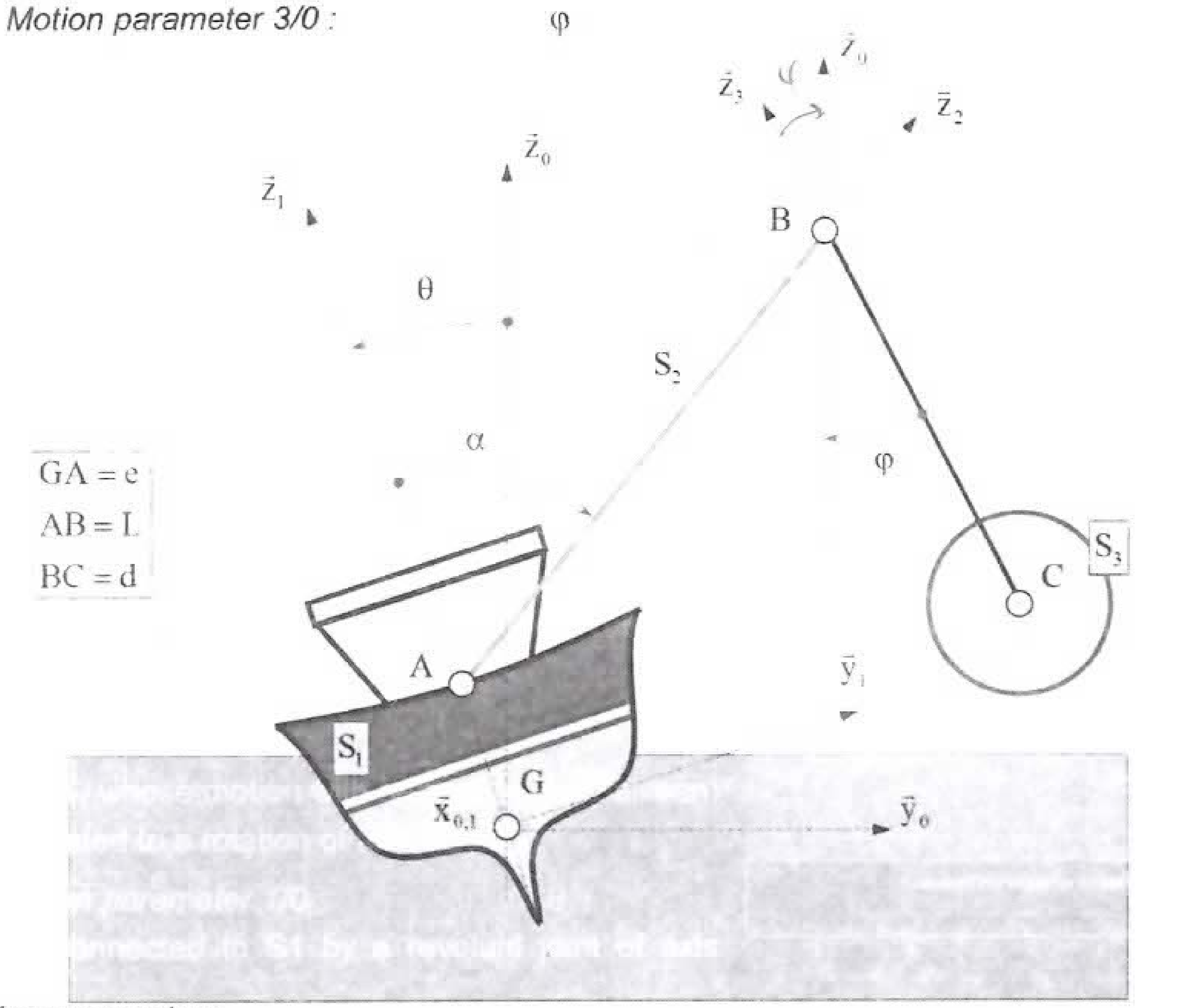
It comprises:

- the ship S_1 whose motion with respect to R_0 (Galilean) is assimilated to a rotation of axis $(G, \vec{x}_{0,1})$

  Motion parameter 1/0 : θ
- arm S_2 connected to S1 by a revolute joint of axis $(A, \vec{x}_{1,2})$

  Motion parameter 2/1 : α
- of the seaweed load S_3 connected S2 by a revolute joint of axis $(B, \vec{x}_{2,3})$

  Motion parameter 3/0 : φ



Mass geometry :

S_1 : centre of mass G, mass m_1 and moment of inertia A_1 with respect to $(G, \vec{x}_1)$

S_2 : centre of mass G_2 (at mid-length of AB), mass m_2 and negligible cross-section.

S_3 : made of a rigid, massless link BC plus a sphere of centre C, radius R and mass m_3

External mechanical actions :

- The swell exerts a periodic torque $M_h \sin(\omega t)\vec{x}_{0,1}$ on S_1
- The **pressure of the water on the shell** combined with **the weight of the ship plus** that of the arm and seaweed $\{S_1 + S_2 + S_3\}$ generate the following force wrench:

$$\{T_{R/1}\} = \begin{cases} \vec{R}_{R/1} = \vec{0} \\ \vec{M}_{R/1}(G) = M_R\,\vec{x}_{0,1} \end{cases}$$

For the sake of simplicity, it is further assumed that the moment varies linearly with the angle of rolling θ such that $M_R = -K\theta$.

Part I – Mass geometry :

Q 1 - Give the matrix of inertia of S_3 at point C and then at point B (in terms of m_3, R and d), identify the moment of inertia I_3 of S_3 with respect to axis $(B, \vec{x}_3)$.

Q 2 - Give the matrix of inertia of S_2 at point A (in terms of m_2 and L).

Q 3 - Assuming that $\alpha = Cste$ (i.e. S_2 does not move with respect to S_1), give the moment of inertia I_Σ of $\Sigma = \{S_1 + S_2\}$ with respect to axis $(G, \vec{x}_1)$.

Part II – Kinetics :

Considering the following hypotheses / simplifications:

- $\alpha = Cste$ S_2 does not move with respect to S_1
- $e = 0$ which implies that $A \equiv G$
- G is fixed in the Galilean frame R_0
- the mass of S_2 is neglected
- S_3 is assimilated to a point (lumped) mass m_3 located at point C

Q 4 - Calculate the sum and moment at G of the Galilean dynamic wrench for $\Sigma = \{S_1 + S_2\}$.

Q 5 - Calculate the sum and moment at B of the Galilean momentum wrench for S_3.

Hint: keep the expressions of the components of $\vec{V}^0(C)$ in R_2 and R_3

Q 6 - Calculate the sum and moment at B of the Galilean dynamic wrench for S_3.

Hint: keep the expressions of the components of $\vec{J}^0(C)$ in R_2 and R_3

Part III – Dynamics :

Using the same hypotheses and simplifications as in Part II

Q 7 - Develop the dynamic moment theorem for $\Sigma_1 = \{S_3\}$ projected on $(B, \vec{x}_{3,2})$.

Develop the dynamic moment theorem for $\Sigma_2 = \{S_1 + S_2 + S_3\}$ projected on $(G, \vec{x}_{0,1})$

indication: do not try to express the dynamic moment of S_3 at point G but use the notation $\vec{\delta}_3^0(G) = \delta_3\,\vec{x}_{0,1}$