Physique : Exam n°1

Thursday 17th October 2013 | Duration: 1h45

Tentitative marking scheme: I: 10 points, II: 10 points

The exam comprises two separate parts. Part I evaluates "assessment objective 1" ("develop a scientific approach to problem solving"). "). Parts II evaluates "objective 2" ("acquire and master enduring knowledge"). Note that he marking scheme is approximate, for indicative purposes only.

Exercise I: Auto-induced forces in a square wire loop

Let us consider a conductive, square-sided wire loop, of centre O and sides of length a, located in plane (xOy), such that its sides are parallel to axes (Ox) and (Oy). A uniform and constant current, $I > 0$, flows through the loop in an anti-clockwise direction. We will study the effects on the sides of the loop which result as a consequence of the generated magnetic field.

To simplify the study, we will assume that each side generates the same magnetic field as an infinite straight wire.

1. Sketch the loop highlighting clearly the axes and the current. Indicate the direction of the magnetic field inside and outside of the loop and justify.

2. Using whichever method you prefer, determine the magnetic field created by an infinite wire of axis $(O'z')$ through which a current I is flowing in the direction of the axis. (Use an appropriate system of coordinates)

3. Deduce the magnetic field created at point M, of coordinates $(\frac{a}{2}; y)$, which belongs to the right hand side of the loop, by the three other sides.

4. Using Biot and Savart, prove that the magnetic field created by the right hand side of the loop does not exert a force on this side.
   Show that the elementary Laplace force which exists on a portion of the right hand side of the loop of length dy, centred at M, is given by:

$$\overrightarrow{dF_L} = k\left[\frac{1}{b} + \frac{1}{c-y} + \frac{1}{c+y}\right] dy\, \vec{u}$$

   where k, b, c and $\vec{u}$ are quantities for which you must give the expressions.

5. Comment on this result. + expression of dF_L

6. Discuss the approximation of using an infinite wire to derive the field created by each of the segments.

Exercise II: Behavior of an RLC series circuit submitted to various excitations

Let's consider the RLC circuit shown in Fig. 1. This circuit is supplied by an ideal voltage source which can deliver different types of signals: continuous, sinusoidal, square wave,...

The voltages are denoted in the following way:
e(t) is the voltage delivered by the generator,
$u_C(t)$ is the voltage across the terminals of the capacitor,
$u_L(t)$ is the voltage across the terminals of the coil and
$u_R(t)$ is the voltage across the terminals of the resistance.

For the numerical applications, please use:

- **C=0.22 μF,**
- **L=10 mH ,**
- **R=40 Ω.**

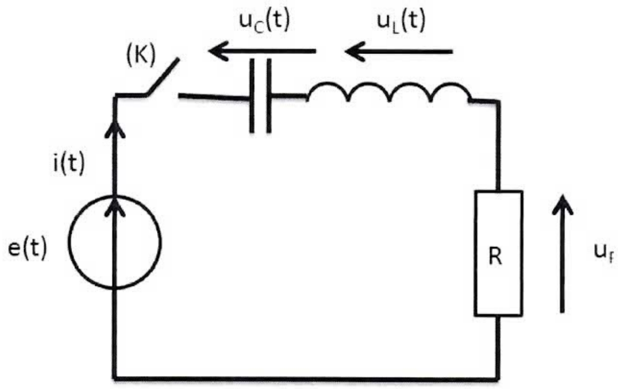

Figure 1 : RLC series circuit

The voltage is applied to the circuit using the switch (K).

The times $t = 0^-$ and $t = 0^+$ correspond respectively to the instants just before and after the closing of the switch, the closing of the switch occurring at the instant t = 0. The capacitor is initially discharged.

Part 1: In this part, the generator supplies a **continuous voltage** and we take e(t) = E = 12 V. We are interested in both transient and steady states.

1- What are the values of the currents $i(0^-)$, $i(0^+)$ and of the voltages $u_C(0^-)$, $u_C(0^+)$, $u_L(0^-)$ and $u_L(0^+)$? Each answer will be carefully justified.

2- What is the steady-state value of the current i?

3- In this circuit set-up, what is the differential equation fulfilled by the current i(t)? Please detail the calculations. Introduce in this equation the damping factor $\delta = \frac{R}{2L}$ and the resonant angular frequencyof the circuit $\omega_0 = \frac{1}{\sqrt{LC}}$. .

4- What are the numerical values of δ and ω_0 ?

5- Among the 3 general solutions given below, justify which one is the correct one:

(1) $i(t) = Ke^{-\delta t}\cos(\sqrt{(\omega_0^2 - \delta^2)}t + \varphi)$

(2) $i(t) = (Kt + K_1)e^{-\delta t}$

(3) $i(t) = K_1 e^{\left(-\delta - \sqrt{\delta^2 - \omega_0^2}\right)t} + K_2 e^{\left(-\delta + \sqrt{\delta^2 - \omega_0^2}\right)t}$

6- How could we obtain the integration constants? (Specify the equations to be considered). Do not try to determine these constants.

7- In figure 2, 2 traces are presented denoted plot 1 and plot 2, respectively. To which voltage, $u_R(t)$, $u_L(t)$ or $u_C(t)$, do each of them correspond ? Justify carefully your answer.