21th November, 2013

MECHANICS - Test 1
SCAN 70-71-72
1h30

Personal formula sheets are authorised
Parts 1, 2 and 3 are independent

Part 1 - Statics

Consider the triangular floor A', B', C' supported by the three legs AA', BB', CC' of equal length a and all in the direction of $\vec{z}$ (Figure 1). Points A, B, C are in the plane z = 0 (ground) and the contacts at A, B and C are point contacts with no friction.
The triangular floor A', B', C' and the three bars AA', BB', CC' form a unique rigid solid.
A'B'C' is equilateral of edge a (and so is triangle A, B, C). The coordinates of A with respect to the coordinate system $(O, \vec{x}, \vec{y}, \vec{z})$ are (0,0,0), those of B (a,0,0) and those of C $(\frac{a}{2}, \frac{a\sqrt{3}}{2}, 0)$
The triangular floor is submitted to a force $-F\vec{z}$ (F positive) passing through point D of coordinates $(\frac{a}{4}, \frac{a}{4}, a)$.
The weights are neglected.

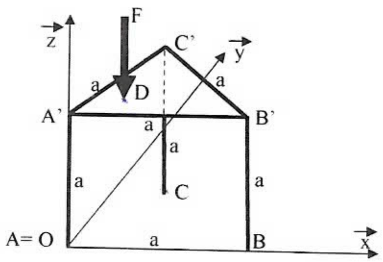

Figure 1

- A1. List of the external actions on the solid using the wrench formalism. Develop the equilibrium equations.
- A2. Calculate the reaction force wrenches at A, B, C in terms of F and a.

Part 2 – Graphical statics

The planar structure in Figure 2 is composed of 4 rigid bars labelled 1,2,3,4 all connected by perfect revolute joints. Bars 1 and 4 are connected to a fixed wall by two perfect revolute joints of axes $(A, \vec{z})$ et $(B, \vec{z})$ respectively. A force $\vec{F}$ of amplitude 2000 N is applied at point H in the downward vertical direction as indicated in Figure 2. The weights of all the bars are neglected.

a) Isolating every bar but also the three-force member corresponding to the system formed by all the bars, determine graphically the mechanical actions $\vec{B}_{0\to 4}$ and $\vec{A}_{0\to 1}$ in the joints at A and B (*hint: use the properties of two-and three-force members*)
b) Then determine the force $\vec{E}_{3\to 1}$ in the joint at E.

Use the annex for the graphical constructions (to be handed in with your paper) and explain your developments (precision and conciseness will be appreciated).

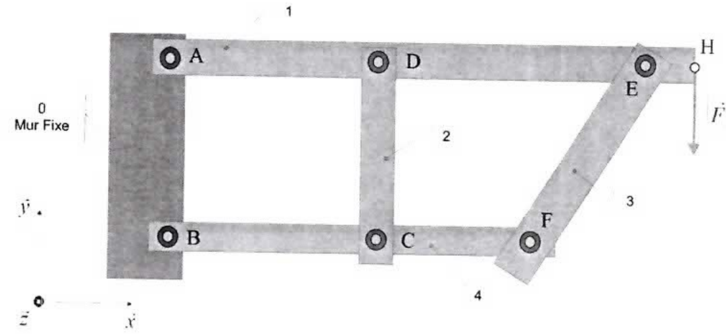

Figure 2 – Planar structure

Part 3 – Study of a robot gripper

The robot gripper shown here =>
is represented by the planar kinematic model shown in Figure 3.

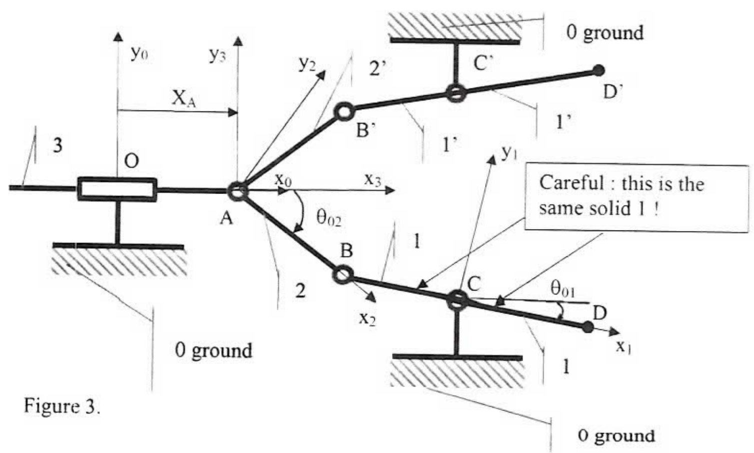

Figure 3.

The gripper being symmetrical, the study will be focused on the lower branch composed of solids 1 and 2 only.

The coordinate system $(O, \vec{x}_0, \vec{y}_0, \vec{z}_0)$ is attached to solid 0 (ground); $\vec{x}_0$ is in the direction of the gripper axis of symmetry (horizontal in Figure 3); $\vec{y}_0$ is the upward vertical direction.

Solid 3 is a bar which is connected to the ground by a prismatic joint of axis $(O, \vec{x}_{0,3})$ and parameter $X_A = \vec{OA}.\vec{x}_{0,3}$ *(input variable for the system)*.

Solid 2 is the bar AB connected to solid 3 by a revolute joint of axis $(A, \vec{z}_{0,2})$ and parameter θ_{02}

Solid 1 is the bar BD connected to the ground (solid 0) by a revolute joint of axis $(C, \vec{z}_{0,1})$ and parameter θ_{01}. Solid 1 is also connected to solid 2 by a revolute joint of axis $(B, \vec{z}_{2,1})$ with no parameter.

The dimensions are such that $\vec{AB} = L_{AB}\vec{x}_2$, $\vec{BC} = L_{BC}\vec{x}_1$, $\vec{CD} = L_{CD}\vec{x}_1$ and $\vec{OC} = X_C\vec{x}_0 + Y_C\vec{y}_0$ (L_{AB}, L_{BC}, L_{CD}, X_C, Y_C *are given*)

Questions:

1. Frame definition, change of basis diagrams and graph of links.

2. Find the constraint equation(s) and deduce the relationship between the input variable X_A and the output variable θ_{01}.

3. A vertical force $-F\vec{y}_0$ of amplitude F=200 N is applied at point D (Figure 4). The system is in equilibrium for the angles θ_{01} = -10° and θ_{02} = -40° and the lengths are L_{AB} = 50 mm, L_{BC} = 46 mm and L_{CD} = 50 mm. All the weights are neglected.

a) Using the particular equilibrium condition for solid 2, propose an interesting coordinate system for expressing the mechanical actions at A and B. Justify.

b) Determine analytically the external forces on solid 1.

c) Determine the internal force wrench in AB

d) Determine the axial force on solid 3 needed to produce a contact force F=200 N on an object when considering the complete gripper

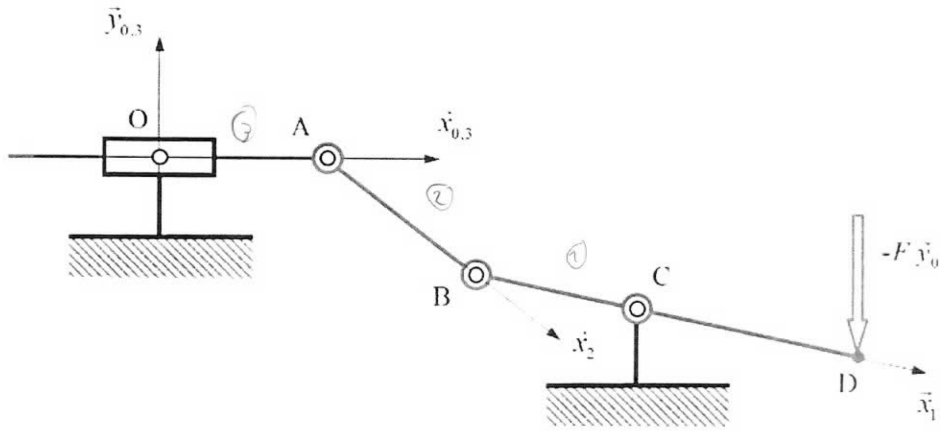

Figure 4