INSTITUT NATIONAL DES SCIENCES APPLIQUÉES DE LYON

**Département du Premier Cycle - SCAN - First**

TEST 2 MTES - DURATION 1 H 30 MIN

**Warnings and advices**
– **Documents, dictionaries phones, and calculators are FORBIDDEN.**
– **The marking scheme is only given as an indication.**

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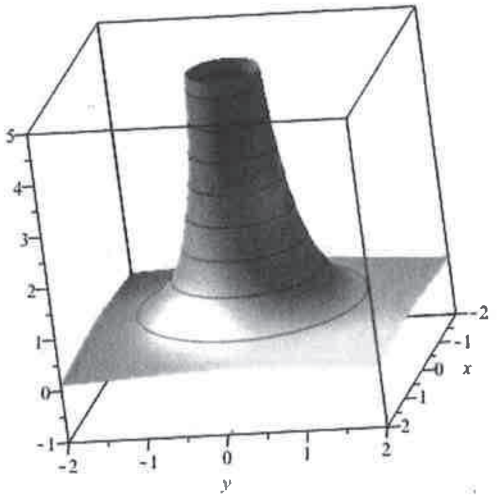
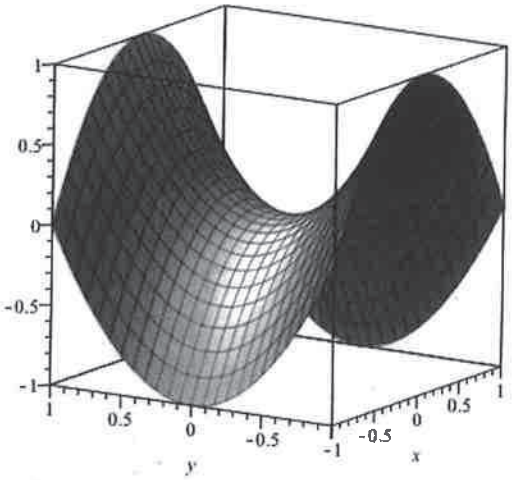
**EXERCICE 1** *3 points*
Here are the graph of two functions $f$ and $g$ from $\mathbb{R}^2$ to $\mathbb{R}$.




graph of f(x, y)




graph of g(x, y)

1. Suppose that $f$ is differentiable on its domain, what is the sign of $\frac{\partial f}{\partial y}$ at the points $(0,-1)$ and $(0,1)$.
2. Suppose that $g$ is differentiable on $\mathbb{R}^2$, can you find a point $(x_0, y_0) \in \mathbb{R}^2$ such that $\frac{\partial g}{\partial y}(x_0, y_0) = 0$. Is it a minimum or a maximum ?
3. Propose a simple expression for the functions $f$ and $g$.
4. Draw the graph of the partial map of $f$ with respect to $y$ at the point $(1, 0)$.
5. Draw the graphs of the partial maps of $g$ at the point $(0, 0)$.

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**EXERCICE 2** *5 points*
We consider the following differential form $\omega$ on $\mathbb{R}^3$ :

$$\omega = (y(1-x) + z^2)dx + xdy - 2zdz$$

1. Show that this differential form $\omega$ is not exact.
2. Find a function $\phi(x)$ such that :
   - $\phi(0) = 1$
   - $\omega_1 = \phi\omega$ is exact.
3. Find $f$ such that $df = \omega_1$.

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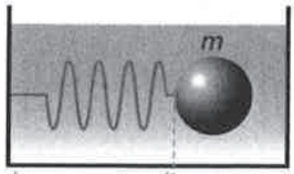
**EXERCICE 3** *6 points*

We consider the following damped linear oscillator in a fluid.




- $m$ is the mass.
- $k$ is the rate constant of the spring.
- $\lambda$ is the coefficient of friction.

We define when $\lambda^2 < 4mk$ :

- $\omega_0(k, m) = \sqrt{\dfrac{k}{m}}$ the proper angular frequency.
- $\gamma(\lambda, m) = \dfrac{\lambda}{m}$ the damping ratio.
- $\omega(\omega_0, \gamma) = \omega_0\sqrt{1 - \dfrac{\gamma^2}{4\omega_0^2}}$ the angular frequency of the pseudo-oscillations.

1. Compute the logarithmic differential $d(\ln(\omega_0))$ and $d(\ln(\gamma))$.
2. Suppose that with time the mass $m$ and the rate constant $k$ are changing. Suppose that the relative variation of $k$ is of $+50\%$. What has to be the relative variation of mass in order for $\omega_0$ to be constant at the first order ?
3. Show that $d\omega = \dfrac{\omega_0^2}{\omega}\dfrac{d\omega_0}{\omega_0} - \dfrac{1}{4}\dfrac{\gamma^2}{\omega}\dfrac{d\gamma}{\gamma}$.
4. Deduce from the first question a simple expression of $d\omega$ with $dk$, $dm$ and $d\gamma$ instead of $d\omega_0$ and $d\gamma$.
5. Compute the uncertainty $\Delta\omega$ with :

$$k = 1N.m^{-1} \quad m = 1g \quad \lambda = 1N.m^{-1}.s \quad \frac{\Delta k}{k} = \frac{\Delta m}{m} = \frac{\Delta\lambda}{\lambda} = 1\%$$

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**EXERCICE 4** *6 points*

1. Draw the graph of the function : $f(x, y) = 3(\sqrt{x^2 + y^2})$.
2. Let $r, \theta$ and $z$ be the cylindrical coordinates.
   Consider the parametric curve :
   $$\begin{cases} r(t) = t \\ \theta(t) = 2\pi t \\ z(t) = 3t \end{cases} \quad t \in \mathbb{R}^+$$
   (a) Express this parametric curve in cartesian coordinates, i. e. compute $x(t), y(t)$ and $z(t)$.
   (b) Show that this parametric curve is on the graph of $f$.
   (c) Let $M(t)$ be the point of coordinates $x(t), y(t)$ and $z(t)$. Write the vector $\overrightarrow{OM}(t)$ in cartesian coordinates and in the cartesian frame.
   (d) What is the velocity $\overrightarrow{v}$ of the point $M(t)$ ?
3. (a) Give the expression of the vectors $\overrightarrow{e_r}(t), \overrightarrow{e_\theta}(t)$ and $\overrightarrow{e_z}(t)$ of the local cylindrical frame at the point $M(t)$.
   (b) Write the vector $\overrightarrow{OM}(t)$ in cylindrical coordinates and in the cylindrical frame.
   (c) Deduce from it the velocity $\overrightarrow{v}$ of the point $M(t)$ in cylindrical coordinates and in the cylindrical frame.
   (d) Is the $\overrightarrow{e_\theta}$ component of $\overrightarrow{v}$ increasing or decreasing ? Can you explain why ?
4. Give an approximate representation of the trajectory of $M(t)$.

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