Part 1 – Complex numbers:

1- Find the $x+iy$ form of:

a) $\dfrac{(1+i)^2}{1-i}$ b) $2e^{i\pi/6}$ c) $e^{2n\pi i}$ d) $\left(\dfrac{\sqrt{2}}{i-1}\right)^{10}$

2- Find the absolute values of

a) $\dfrac{25}{3+4i}$ b) $\left(\dfrac{1+i}{1-i}\right)^5$

3- Solve $z^2+2z+1-i=0$

4- Prove that $\sum_{k=0}^{n} e^{i(t+k\delta)} = \dfrac{e^{it}\left(1-e^{i(n+1)\delta}\right)}{1-e^{i\delta}}$ and deduce the expression of

$\sin(t)+\sin(t+\delta)+\sin(t+2\delta)+\ldots+\sin(t+n\delta)$ =

5- Introducing $\hat{\theta}$, a complex angle, and using the equation $\sin\hat{\theta} = \dfrac{e^{i\hat{\theta}}-e^{-i\hat{\theta}}}{2i}$ (*admit this result*),

find the correct value of $\sin\left(\dfrac{\pi}{2}+i\ln 2\right)$

a) $\cos i$ b) 3/2 c) $\dfrac{3-i}{2+i}$ d) 5/4 e) $e^{i\ln 2}$

Part 2 – Vectors:

1- a) Find a unit normal vector to the plane containing the three points A $A(-1,1,1), B(2,3,0), C(0,1,-2)$

b) Find the distance between point $O(0,0,0)$ and the plane defined above

c) Find the Cartesian equation of the plane using scalar product.

2- Consider the line through $A(4,-1,3))$ and collinear with $\vec{u} = \vec{x} - 2\vec{y}$, find the distance from point $B(3,2,5)$ and this line

3- Find the projection of vector $\vec{a} = 4\vec{x} - 3\vec{y} + \vec{z}$ on the line passing through points $A(2,3,-1)$ and $B(-2,-4,3)$

4- A person walks 57 meters at an angle of 47° from the x-axis, then turns at an angle of -15° from the x-axis and walks 72 meters, and finally turns at an angle of 30° from the y-axis and walks 24 meters (*angle signs are defined using the classic convention in trigonometry*).

a- Sketch the corresponding path

b- Using vectors, determine how far the person is from the initial position and at what angle from the x-axis.